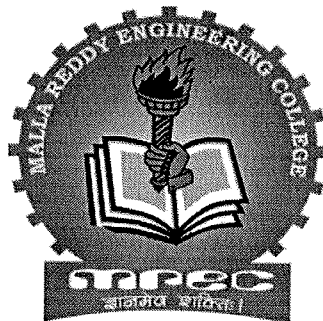
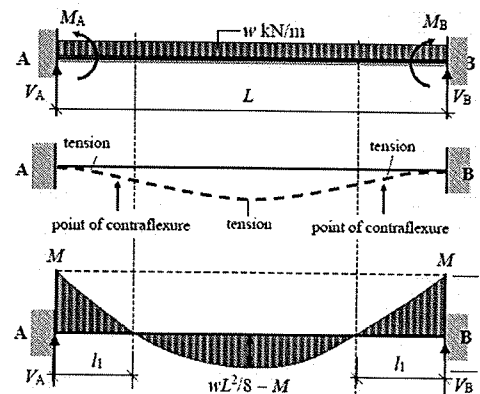
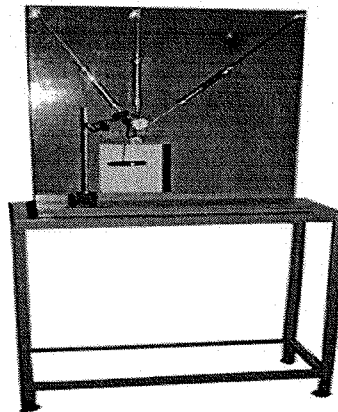
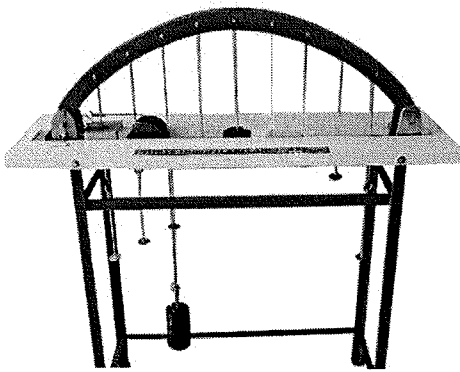


STRUCTURAL ANALYSIS

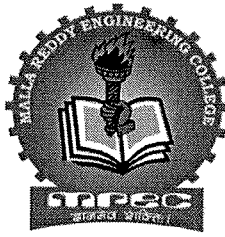
LAB MANUAL



MALLA REDDY ENGINEERING COLLEGE (AUTONOMOUS)

(An Autonomous Institution approved by UGC and affiliated to JNTUH, Approved by AICTE, Accredited by NAAC with 'A' Grade and NBA)

Maisammaguda, Dhulapally (Post. Via. Kompally), Secunderabad – 500 100.



**LAB OBSERVATION
FOR
STRUCTURAL ANALYSIS**

(II YEAR- II SEMESTER)

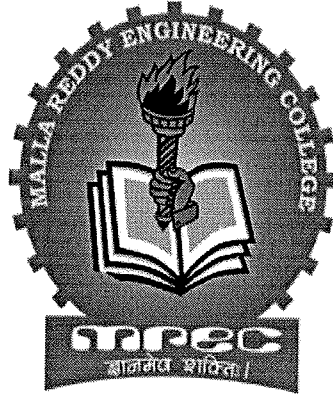
**UNDER GRADUATE
DEPARTMENT OF CIVIL ENGINEERING**



MALLA REDDY ENGINEERING COLLEGE (AUTONOMOUS)

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Maisammaguda, Dhulapally (Post. Via. Kompally), Secunderabad – 500 100.



STRUCTURAL ANALYSIS

STUDENT NAME: -----

ROLL NUMBER: -----

YEAR: -----

STAFF LAB INCHARGE:

CE-HOD

EXTERNAL EXAMINER

PREPARED BY: *B. VAMSI KRISHNA, Assistant Professor, MREC (A)*

LIST OF EXPERIEMNTS

<u>NAME OF THE EXPERIMENT</u>	<u>PAGE.NO</u>
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➤ To know the Flexural Rigidity (EI) of a given beam	
2. MAXWELL-BETTI'S LAW	08
➤ Verification of Maxwell-Betti's Law	
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➤ To study three hinged arch	
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➤ To study two hinged arch	
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➤ Study of behavior of columns and struts with different end conditions.	

2017-18 Onwards (MR-17)	MALLA REDDY ENGINEERING COLLEGE (Autonomous)											B.Tech. VII Semester		
Code: 70131	STRUCTURAL ANALYSIS LAB											L	T	P
Credits: 2												-	-	4

Prerequisites: Structural Analysis-I & II, Strength of Materials, Engineering Mechanics

Course Objective: To impart knowledge on testing of beams, columns, trusses and frames.

List of Experiments:

1. Determination of Flexural Rigidity (EI) of a given beam.
2. Verification of Maxwell-Betti's Law.
3. Experiment on three hinged arch.
4. Experiment on two hinged arch.
5. Verification of moment area theorem for slope and deflection of a given beam.
6. Deflection of a statically determinate pin jointed truss.
7. Forces in members of redundant frames.
8. To find deflection of curved members.
9. Unsymmetrical bending of a cantilever beam.
10. Deflection of fixed beam and influence line for reactions.
11. Deflection studies for a continuous beam and influence line for reactions.
12. Study of behavior of columns and struts with different end conditions.

Course Outcomes:

At the end of the course, students will be able to

1. Understand the concept of Maxwell Theorem and its application.
2. Analyse two hinged and three hinged arches.
3. Analyse trusses and beams curved in plan.
4. Calculate the deflection of fixed and continuous beams due to various types of loading.
5. Study the behaviour of columns and struts.

CO- PO-PSO Mapping (3/2/1 indicates strength of correlation) 3-Strong, 2-Medium, 1-Weak															
CO' S	Programme Outcomes(POs)												PSOs		
	PO 1	PO 2	PO 3	PO 4	PO 5	PO 6	PO 7	PO 8	PO 9	PO 10	PO 11	PO 12	PSO 1	PSO 2	PSO 3
CO 1	3	3	3		2		2		2	2		2	3		
CO 2	3	3	3	1	2				2	2		2	3		
CO 3	3	3	3	2	2				2	2		2	3		
CO 4	3	3	3	3	2				2	2		2	3		
CO 5	3	3	3		2				2	2		2	3		

ANALYSIS ON ENGINEERING MATERIALS

**(Mild Steel, Stainless Steel, Brass,
Copper, Aluminium)**

EXPERIMENT NO :

DATE:

1. FLEXURAL RIGIDITY (EI)

Aim: - To find the value of flexural rigidity (EI) for a given beam and compare it with theoretical value.

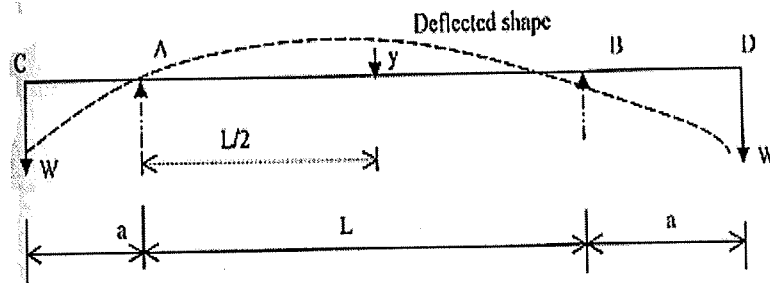
Apparatus: - Elastic Properties of deflected beam, weight's, hanger, dial gauge, scale, and Verniar caliper.

Formula: - (1) Central upward deflection, $y = w.a.L^2 / 8EI$ (1)

(2): $EI = w.a. L^2 / 8y$ (2)

(3) Also it is known that EI for beam = $E \times bd^3 / 12$ (3)

Diagram:-



Theory:- For the beam with two equal overhangs and subjected to two concentrated loads W each at free ends, maximum deflection y at the centre is given by central upward deflection.

Central upward deflection, $y = w.a.L^2 / 8EI$

Where,

a = length of overhang on each side

W = load applied at the free ends

L = main span

E = modulus of elasticity of the material of the beam

I = moment of inertia of cross section of the beam

$EI = w.a. L^2 / 8y$

It is known that, EI for beam = $E \times bd^3 / 12$

Where, b = width of beam

d = depth of beam

Procedure: -

- i) Find b and d of the beam and calculate the theoretical value of EI by Eq. (3).
- ii) Measure the main span and overhang span of the beam with a scale.
- iii) By applying equal loads at the free end of the overhang beam, find the central deflection y.
- iv) Repeat the above steps for different loads.

Observation: -

- 1) Length of main span, L (cm) =
- 2) Length of overhang on each side, a (mm) =
- 3) Width of beam, b (mm) =
- 4) Depth of beam, d (mm) =
- 5) Modulus of elasticity, E (kg/cm²) = $2 \times 10^5 \text{ N/mm}^2$

Observation Table:-

Sr. No.	Equal loads at the two ends (kg)	Dial gauge reading at the midspan of beam (mm)	EI from Eq. (3)	EI from Eq (2)

Calculation: - Average values of EI from observation =mm⁴
 Average values of EI from calculation =mm⁴

Result:- Flexural rigidity (EI) is found same theoretically and experimentally.

Precaution : - Measure the center deflection y very accurately.
 Ensure that the beam is devoid of initial curvature.
 Loading should be within the elastic limit of the materials.

EXPERIMENT NO :

DATE:

2. MAXWELL-BETTI'S LAW

AIM: To find young's modulus of the material of the given beam by conducting bending test on simply supported beam using Maxwell's law of reciprocal deflections.

APPARATUS:

Beam supports, Loading yoke, Slotted weight hanger, Slotted weights. Dial gauge, Dial gauge stand, Scale & Vernier callipers

FORMULA:

For a simply supported beam with concentrated load at mid-span the formulae of deflection is as follows:

$$\delta = \frac{11 WL^3}{768 EI}$$

PROCEDURE:

1. The breadth and depth of the beam along the span is measured and average values are taken.
2. The load is applied in increments and the corresponding deflections with the help of dial gauge are measured.
3. Precautions are to be taken to keep the dial gauge in correct position to measure the desired deflection.
4. The deflections corresponding to various loads for each case are tabulated.
5. The beam is placed horizontally and in the first case, the loads are acted in the middle and dial gauge is placed at 1/4th of the beam and loads are added slowly and according to the load, the readings are noted. Similarly note down the deflections while unloading.
6. In the second case load is placed at 1/4th of the beam and dial gauge at the centre and the readings are noted similar to the first case.

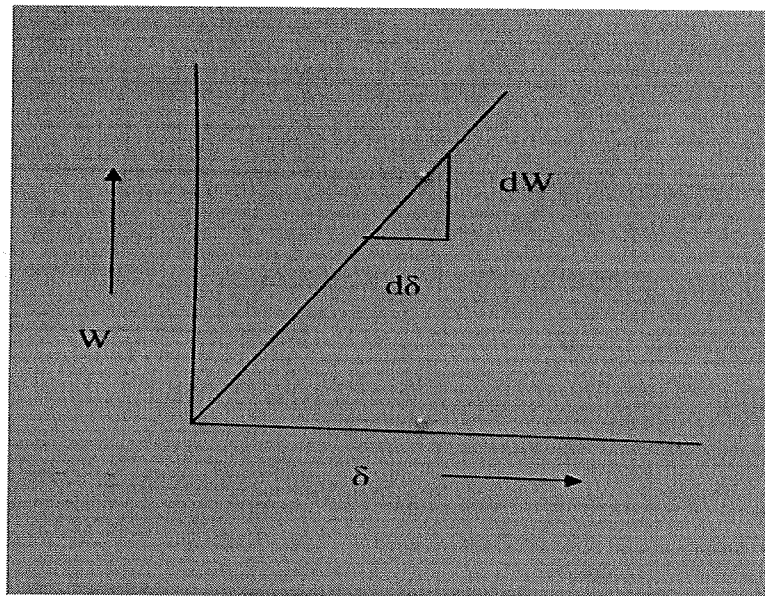
Table 1:

S.No	Parameters of set-up	value
1	Width of the beam (rectangle) cross-section, b mm	
2	Depth of the beam (rectangle) cross-section, d mm	
3	Moment of inertia , bd /12	

CALCULATIONS:

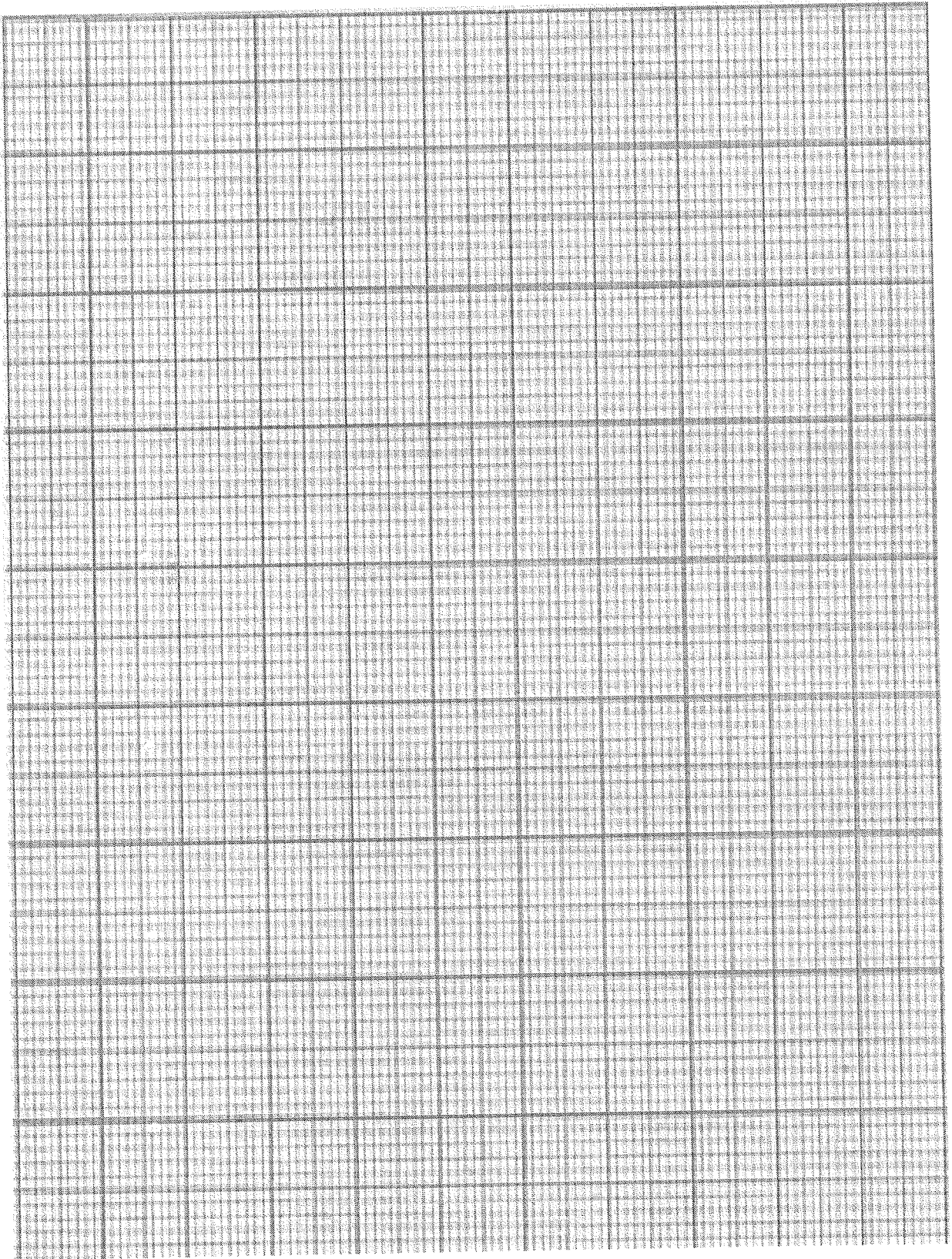
Moment of inertia (I) = mm⁴

Young's Modulus (E) = $(11/768) * (W/6) * (L^3/I)$

GRAPHS TO BE DRAWN:**RESULT:**

The Young's modulus of steel by Maxwell's reciprocal theorem is:

The percentage error is:



EXPERIMENT NO :

DATE:

4. TWO HINGED ARCH

Aim: - To study two hinged arch for the horizontal displacement of the roller end for a given system of loading and to compare the same with those obtained analytically.

Apparatus: - Two Hinged Arch Apparatus, Weight's, Hanger, Dial Gauge, Scale, Vernier Caliper.

Formula: - $\Delta = \frac{5WL}{8r} (a - 2a^3 + a^4)$

Where,

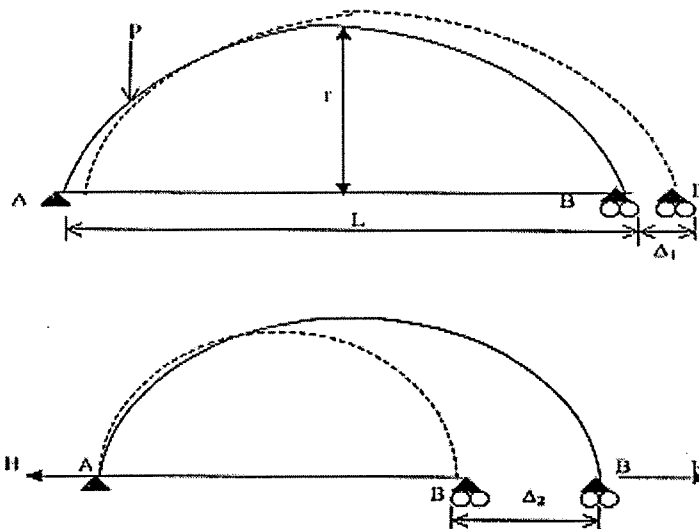
W= Weight applied at end support.

L= Span of two hinged arch.

r= rise of two hinged arch.

a = dial gauge reading.

Diagram:-



Theory :-The two hinged arch is a statically indeterminate structure of the first degree. The horizontal thrust is the redundant reaction and is obtained by the use of strain energy methods. Two hinged arch is made determinate by treating it as a simply supported curved beam and horizontal thrust as a redundant reaction. The arch spreads out under external load. Horizontal thrust is the redundant reaction is obtained by the use of strain energy method.

Procedure: -

- 1) Fix the dial gauge to measure the movement of the roller end of the model and keep the lever out of contact.
- 2) Place a load of 0.5kg on the central hanger of the arch to remove any slackness and taking this as the initial position, set the reading on the dial gauge to zero.

- 3) Now add 1 kg weights to the hanger and tabulated the horizontal movement of the roller end with increase in the load in steps of 1 kg. Take the reading up to 5 kg load. Dial gauge reading should be noted at the time of unloading also.
- 4) Plot a graph between the load and displacement (Theoretical and Experimental) compare. Theoretical values should be computed by using horizontal displacement formula.
- 5) Now move the lever in contact with 200gm hanger on ratio 4/1 position with a 1kg load on the first hanger. Set the initial reading of the dial gauge to zero.
- 6) Place additional 5 kg load on the first hanger without shock and observe the dial gauge reading.
- 7) Restore the dial gauge reading to zero by adding loads to the lever hanger, say the load is w kg.
- 8) The experimental values of the influence line ordinate at the first hanger position shall be $\frac{4w}{5}$.
- 9) Repeat the steps 5 to 8 for all other hanger loading positions and tabulate. Plot the influence line ordinates.
- 10) Compare the experimental values with those obtained theoretically by using equation. (5).

Observation Table:-

Table: - 1
Horizontal displacement

Sr.No.	Central load (kg)	0.0	1.0	2.0	3.0	4.0	5.0	6.0
	Observed horizontal Displacement (mm)							
	Calculated horizontal Displacement Eq. (4)							

Sample Calculation: - Central load (kg) =.....

$$\begin{aligned} \text{Observed horizontal Displacement (mm).} &= \\ \text{Calculated horizontal Displacement} &= H = \frac{5WL}{8r} (a - 2a^3 + a^4) \\ &= \dots\dots\dots \end{aligned}$$

Result :-The observed and horizontal displacement is nearly same.

Precaution :- Apply the loads without jerk.

:- Perform the experiment away from vibration and other disturbances.

EXPERIMENT NO :

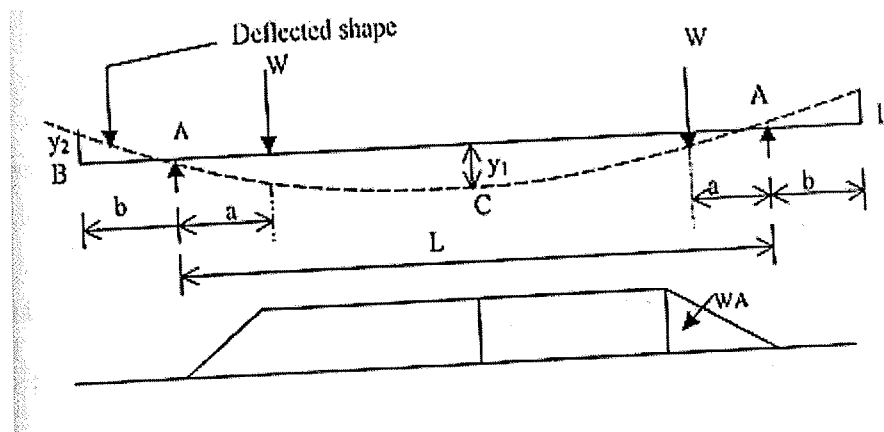
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5. MOMENT AREA THEOREM

Aim: - To verify the moment area theorem regarding the slopes and deflections of the beam.

Apparatus: - Moment of area theorem apparatus.

Diagram:-



Theory :-

According to moment area theorem

1. The change of slope of the tangents of the elastic curve between any two points of the deflected beam is equal to the area of M/EI diagram between these two points.
2. The deflection of any point relative to tangent at any other point is equal to the moment of the area of the M/EI diagram between the two point at which the deflection is required.

$$\text{Slope at B} = Y_2 / b$$

Since the tangent at C is horizontal due to symmetry,

$$\text{Slope at B} = \text{shaded area} / EI = 1 / EI [Wa^2 / 2 + Wa (L/2 - a)]$$

Displacement at B with respect to tangent at C

$$= (y_1 + y_2) = \text{Moment of shaded area about B} / EI$$

$$= 1 / EI [Wa^2 / 2 (b + 2/3a) + Wa (L/2 - a)(b + a/2 + L/4)]$$

Procedure: -

1. Measure a, b and L of the beam
2. Place the hangers at equal distance from the supports A and load them with equal loads.
3. Measure the deflection by dial gauges at the end B (y_2) and at the center C (y_1)
4. Repeat the above steps for different loads.

Observation Table:-

Length of main span, L (mm) =

Length of overhang on each side, a (mm) = Modulus

of elasticity, E (N/mm^2) = 2×10^5

Sl. No.	Load at each Hanger (N)	Deflection at Free end y_2 (cm)	Deflection at Free end Theoretically	Slope at B Y_2 / b	Slope at B Theoretically

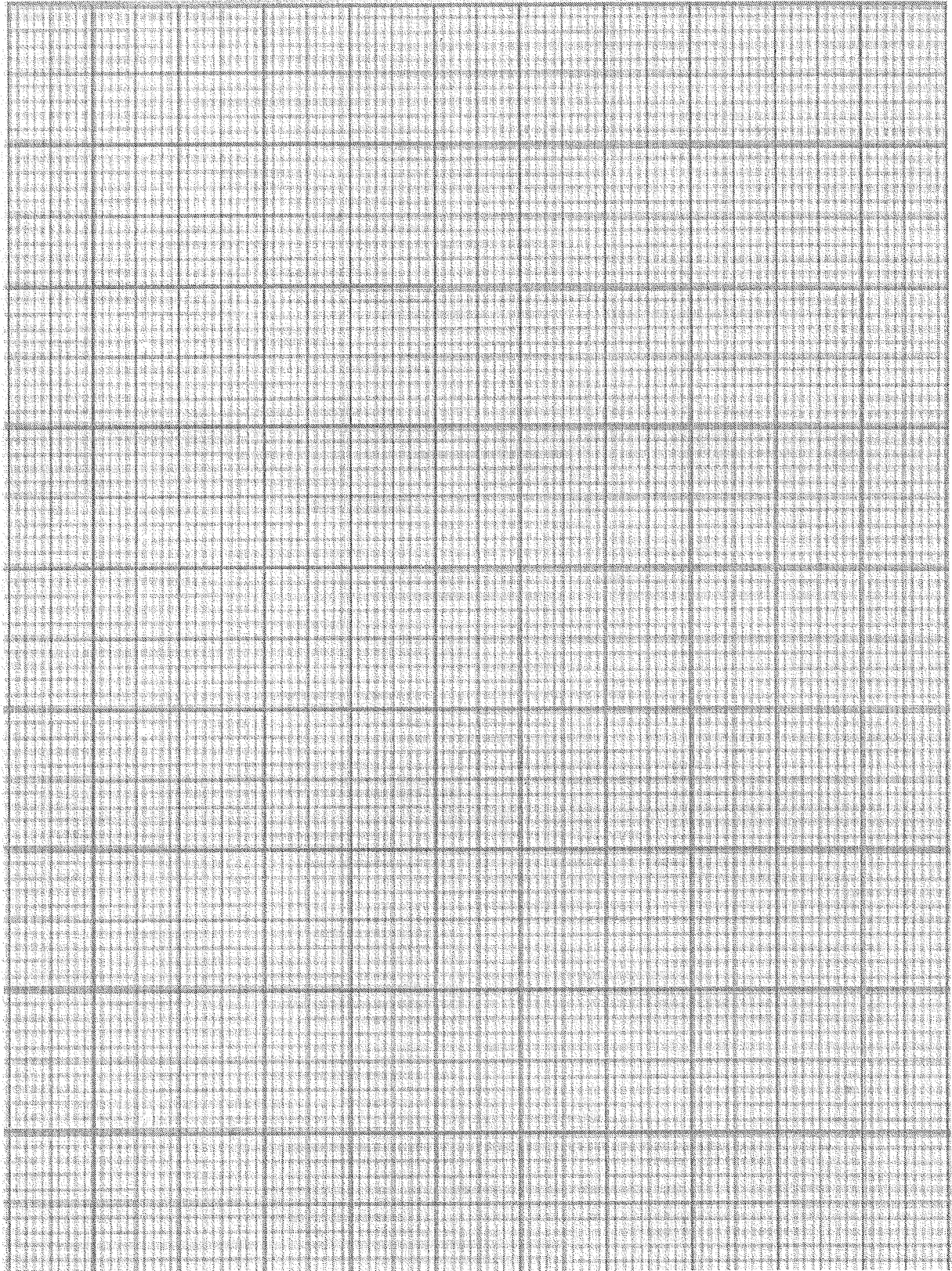
Calculation:-

1. Calculate the slope at B as y_2 / b (measured value).
2. Compute slope and deflection at B theoretically from B.M.D. and compare with experimental values.
3. Deflection at C = y_1 (measured value).
4. Deflection at C = Average calculated value

Result :- The slope and deflection obtained is close to the slope and deflection obtained by using moment area method.

Precaution:-

- .Apply the concentration loads without jerks.
- .Measures the deflection only when the beam attainsion.
- .Measures the deflection very care fully and accurately.
- . Check the accuracy and least count of dial gauges used for measuring deflections.



EXPERIMENT NO :

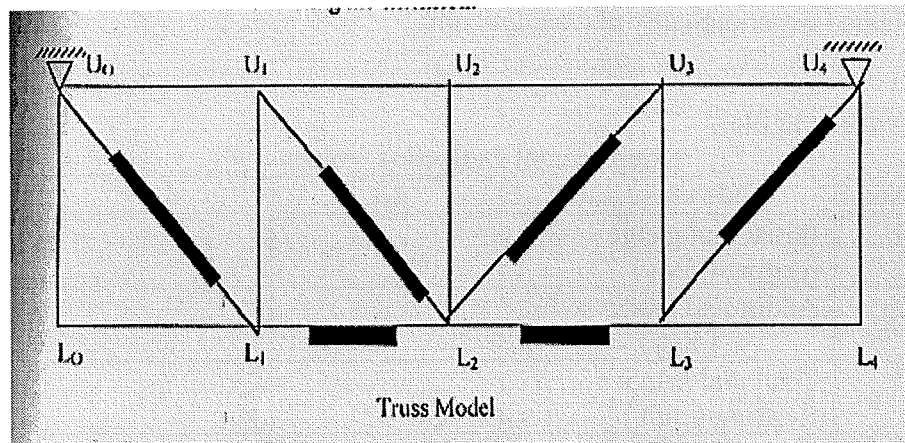
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6. PIN JOINTED TRUSS

Aim : - To determine the deflection of a pin connected truss analytically & graphically and verify the same experimentally.

Apparatus: - Truss Apparatus, Weight's, Hanger, Dial Gauge, Scale, Verniar caliper.

Diagram:-



Theory :-The deflection of a node of a truss under a given loading is determined by:

$$\delta = \sum (TUL/AE)$$

Where, δ = deflection at the node point.

T = Force in any member under the given loading.

U = Force in any member under a unit load applied at the point where the deflection is required. The unit load acts when the loading on the truss have been removed and acts in the same direction in which the deflection is required.

L = Length of any member.

A = Cross sectional area of any member.

E = Young's modulus of elasticity of the material of the member.

Here, (L/AE) is the property of the member, which is equal to its extension per unit load. It may be determined for each member separately by suspending a load from it and noting the extension.

Procedure: -

- 1) Detach each spring from the member. Plot extension against load by suspending load from the spring and noting the extension. From the graph, obtain the extension per unit load (stiffness).
- 2) For initial position of the truss, load each node with 0.5 kg load to activate each member. Now place the dial gauges in position for measuring the deflections and note down the initial reading in the dial gauges.

- 3) Also put additional load of 1kg, at L1, 2kg, L2, and 1kg at L3, and note the final reading in the dial gauges. The difference between the two readings will give the desired deflection at the nodal points. Central deflection y.
- 4) Calculate the deflection for the three nodes L1, L2, and L3 from the formula given in Eq. (1) and compare the same with the experimental values obtained in step 3.
- 5) Draw the Willot – Mohr diagram for deflection and compare the deflection so obtained experimentally and analytically.

Observation Table:-

Experimental Deflection Values

S.No.	Node Deflection	L1	L2	L3
1	Initial dial gauge reading (mm)			
2	Additional loads (kgs)			
3	Final dial gauge Reading (mm)			
4	Deflection (3) – (1) (mm)			

Analytical Calculation

Member	L/AE	F (kg)	Node L ₁		Node L ₂		Node L ₃	
			U (kg)	FUL/AE	U (kg)	FUL/AE	U (kg)	FUL/AE
U ₀ U ₁								
U ₁ U ₂								
U ₂ U ₃								
U ₃ U ₄								
L ₀ L ₁								
L ₁ L ₂								
L ₂ L ₃								
L ₃ L ₄								
U ₀ L ₀								
U ₁ L ₁								
U ₂ L ₂								
U ₃ L ₃								
U ₄ L ₄								
U ₀ L ₁								
U ₁ L ₂								
U ₃ L ₂								
U ₄ L ₃								

Sample Calculation: - Member =

$$L/AE = \dots\dots\dots$$

Analytical deflection:= FUL/AE

Result :-The theoretical and experimental deflection in various members is found same.

Precaution : - i) Apply the loads without any jerk.

- ii) Measure the deflection to the nearest of a millimeter.
- iii) Perform the experiment at a location, which is away from any
- iv) External disturbance.
- v) Ensure that the supports are rigid.

EXPERIMENT NO :

DATE:

7. REDUNDANT FRAMES

Object: Experimental and analytical study of a 3 bar pin jointed truss.

Theory:-

Following notation will be used:-

L_1 = length of member AD

L_2 = length of member CD

L_3 = length of member BD

a = distance AB

b = distance BC

w = applied load W at D

V = vertical displacement of D under an applied load W.

U = horizontal displacement of D under an applied load W.

T_1 = tensile force in member AD

T_2 = Tensile force in member CD T_3 = Tensile force in member BD

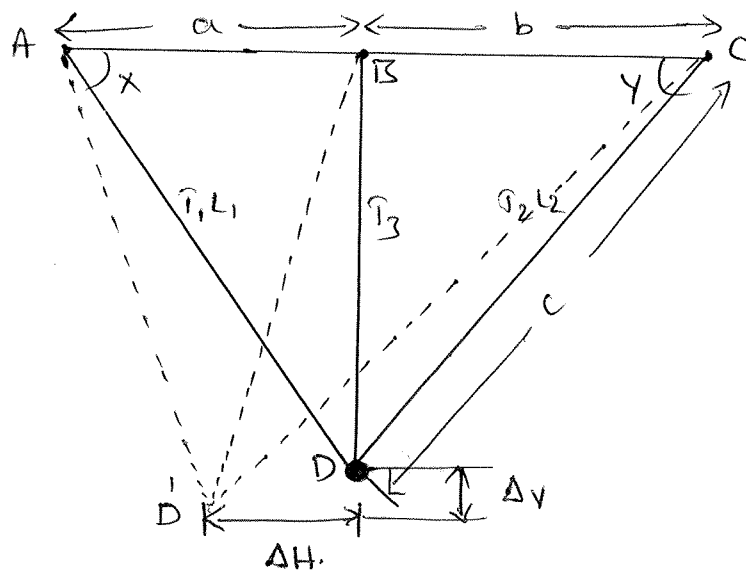
The values of U, V, T_1 , T_2 and T_3 are given by the following expressions:-

AIM:- Comparison of Experimental and Theoretical results of forces in the members and displacement of the loaded joint D. of a Three Bar suspension system for vertical loads.

APPARATUS:- Consists of Three suspension members (spring balances) of different stiffness which are jointed at a point to form the redundant joint. The upper end of the member tied in position to the vertical board of the table.

- Vertical load is applied at the joint and Horizontal (H) and Vertical (V) displacement is noted on the paper fixed at the joint point.
- Force is noted on the Spring balance (T_1 , T_2 & T_3)

LOADING DIAGRAM :-



Three Bar Suspension System

The theoretical calculation for the above is as follows:-

$$T_1 = \frac{(CV - a\Delta H) \cdot K_1}{L_1}$$

$$T_2 = \frac{(CV - a\Delta H) K_2}{L_2}$$

$$T_3 = K_3 \cdot \Delta V$$

$$\text{Horizontal Movement } \Delta H = \frac{W}{C} \left[\frac{N_1 a - N_2 b}{N_1 N_2 (a+b)^2 + N_3 (N_1 a^2 + N_2 b^2)} \right]$$

$$\text{Vertical Movement } \Delta V = \frac{W}{C^2} \left[\frac{N_1 a^2 - N_2 b^2}{N_1 N_2 (a+b)^2 + N_3 (N_1 a^2 + N_2 b^2)} \right]$$

$$N_1 = \frac{K_1}{L_1^2}$$

$$N_2 = \frac{K_2}{L_2^2}$$

$$N_3 = \frac{K_3}{L_3^2}$$

\hat{P}_1 = Force in member AD

\hat{P}_2 = Force in member CD

\hat{P}_3 = Force in member BD

K = Stiffness of the Spring

OBSERVATION :

→ Measure the distance a, b, c, L_1 & L_2 with Scale which will be standard values for the Apparatus.

$a =$ _____ mm

$b =$ _____ mm

$c =$ _____ mm

$L_1 =$ _____ mm

$L_2 = \text{————— mm.}$

→ For initial tension in the member load 0.5 Kg and take it as the datum.

→ Note down the deflection ΔH & ΔV from paper trace and forces T_1 , T_2 & T_3 for different set of loads (Experimental value).

TABULAR COLUMN

Load (Kg)	ΔV (mm)		ΔH (mm)		T_1 (Kg)		T_2 (Kg)		T_3 (Kg)	
	Obse	Calcu	obser	Calcu	ob	ca	ob	ca	ob	ca

RESULTS:

Compare the Experimental and Analytical values.

T_1 , T_2 & T_3

EXPERIMENT NO :

DATE:

8. CURVED BEAMS

AIM:- TO Determine the Elastic displacement of the Curved members Experimentally and verify the Same with theoretical value.

APPARATUS:- Consists of Steel bar of Standard Cross section which is used to make different Curved members with Circle, Semi circle, Quadrant & Quadrant of Circle with Straight Arm. The Bottom Ends of the members are fixed to the base. Under the application of load at free End its deflections measured with Dial gauge.

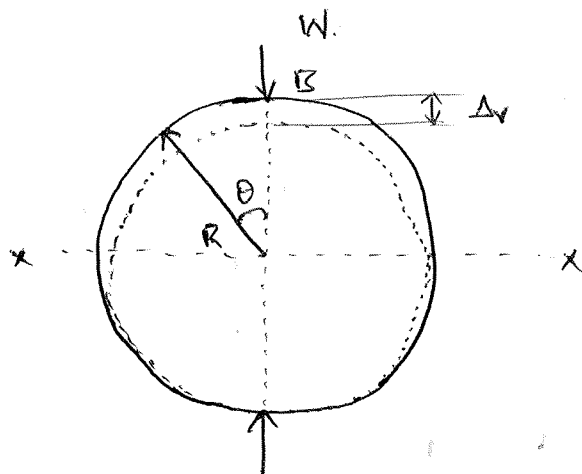
THEORY:- Castigliano's first theorem is used to find the Elastic displacements of Curved members.

Theorem States "Partial derivative of the total Strain Energy of a Structure with respect to any force gives the displacement of the point its application in the direction of the force."

In all cases the horizontal Δ_H and Vertical Deflection Δ_V due to vertical load V are to be determined. These deflections are obtained by using Castigliano's theorem where Strain Energy

due to bending only is taken in to account. The results obtained for the four curved members shall be as follows.

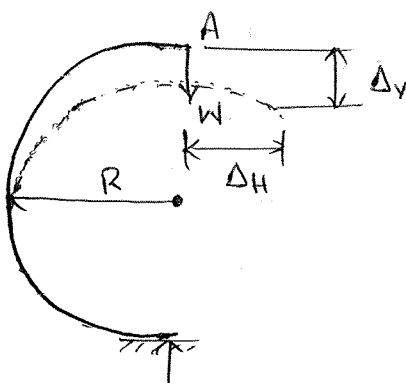
(i) CIRCLE LOADING DIAGRAM



Vertical Displacement of loaded point B

$$\Delta_{BV} = \frac{WR^3}{4\pi EI} (\pi^2 - 8)$$

(ii) SEMI CIRCLE LOADING DIAGRAM



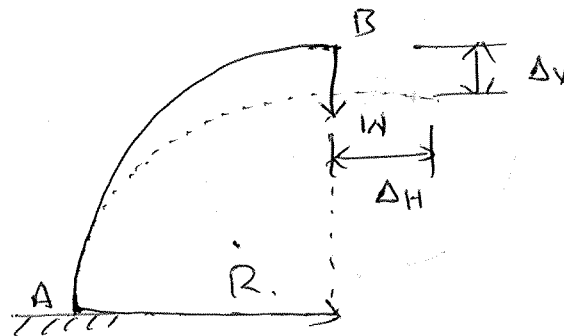
Vertical Displacement

$$\Delta_V = \frac{W \pi R^2}{6EI}$$

Horizontal Displacement

$$\Delta_H = \frac{WR^3}{EI}$$

(ii) QUADRANT OF A CIRCLE



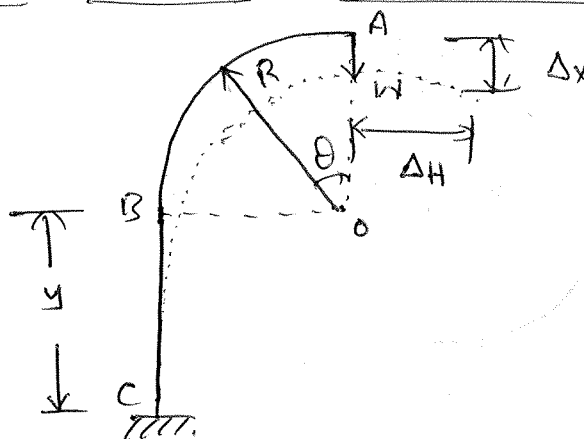
Vertical Displacement

$$\Delta_{BV} = \frac{\pi WR^3}{4EI}$$

Horizontal Displacement

$$\Delta_{BH} = \frac{WR^3}{2EI}$$

(iv) QUADRANT WITH A STRAIGHT LEG



Vertical Displacement of load

$$\Delta_v = \frac{\pi W R^3}{4 E I} + \frac{W R^2 y}{E I}$$

Horizontal Displacement of load

$$\Delta_H = \frac{W R}{2 E I} (R + y)^2$$

PROCEDURE :-

- Place load on Hanger to activate member and treat this as initial position for measuring Deflections.
- Fix the dial gauges for measuring horizontal and vertical deflections.
- place the additional loads in steps as in the tabulation and note down the load and Dial Gauge reading.

OBSERVATIONS :-Width of section (mm) $b =$ Depth of section (mm) $d =$ Least Moment of Inertia $I = \frac{bd^3}{12} =$ mm⁴Youngs Modulus $E = 2 \times 10^5$ N/mm²

TABULAR COLUMNS

S.NO	Load (kg)	Dial gauge Reading	Deflection (mm)

CIRCLE

SEMI CIRCLE

S.NO	Load (kg)	Dial gauge Reading		Deflection (mm)	
		Δ_v	Δ_H	Δ_v	Δ_H

QUADRANT

S.NO	Load (kg)	Dial gauge Reading		Deflection (mm)	
		Δ_v	Δ_H	Δ_v	Δ_H

QUADRANT WITH STRAIGHT ARM.

S.NO	Load (kg)	Dial gauge Reading		Deflection (mm)	
		Δ_v	Δ_H	Δ_v	Δ_H

- RESULT :
- PLOT THE GRAPH LOAD VS DEFLECTION.
 - COMPARE THE DEFLECTIONS EXPERIMENTALLY & THEORETICALLY

EXPERIMENT NO :

DATE:

9. UNSYMMETRICAL BENDING

Object:- Experimental and analytical study of deflections for unsymmetrical bending of a cantilever beam.

Theory:-

In structural members subjected to flexure, the Euler Bernoulli's equation $\{\sigma/y = M/I = E/R\}$ is valid only if the applied bending moment acts about one or the other principal axis of the cross section. However a member may be subject to a bending moment which acts on a plane inclined to the principal axis (say). This type of bending does not occur in a plane of symmetry of the cross section, it is called unsymmetrical bending. Since the problem related to flexure in general differs from symmetrical bending, it may be termed as skew bending.

Every cross-section of a symmetric section has two mutually perpendicular principal axes of inertia, about one of which the moment of inertia is the maximum and about the other a minimum. It can be shown that the symmetric axis of cross-section is one of the principal axes and one at right angles to the same will be the other principal axis.

From the principal of mechanics, any couple which may cause bending moment at a section of a beam may be resolved in to two components. The component of the bending moment acting around x is $M \cos \alpha$, while the one acting around the y-axis is $M \sin \alpha$. The sense of each component follows from the sense of the total moment M. These moments may be used separately in the usual flexure formula and the compound normal stresses follow by super position as follows:

$$\sigma = \pm M_{xx} \cdot y / I_{xx} \pm M_{yy} \cdot x / I_{yy} \dots \dots \dots (i)$$

One or the other of the principal axes of the cross-section.

- (a) Bending Moment in a plane that is not coincident with either of the principal axes.
- (b) Components of the bending moment in the plane of the principal axes.
- (c)

For beams having unsymmetrical cross-section such as an angle (L) or a channel (I) section, if the plane of loading is not coincident with or parallel to one of the principal axes, the bending is not simple. In that case it is said to be unsymmetrical or non-uniplanar bending.

In the present experiment for a cantilever beam of an angle section, the plane of loading is always kept vertical and the angle iron cantilever beam itself is rotated through angles in steps of 45°.

Consider the position of the angle section as shown in Fig. (2). The plane of loading makes an angle ϕ with V-V axis of the section, which is one of the principal axes of the section. The components of the vertical load P along V-V and U-U axes are $P \cos \phi$ and $P \sin \phi$ respectively. The deflections U and V along U-V and V-V axes respectively are given by

$$\Delta_y = P \sin \phi \cdot L^3 / 3EI_{vv} \dots\dots\dots (ii)$$

$$\Delta_x = P \cos \phi \cdot L^3 / 3EI_{uu} \dots\dots\dots (iii)$$

And the magnitude of resultant deflection Δ_{00} , is given by

$$\Delta = \sqrt{\Delta_x^2 + \Delta_y^2} \dots\dots\dots (iv)$$

And its direction is given by

$$\beta = \tan^{-1} \Delta_x / \Delta_y \dots\dots\dots (v)$$

Where β is the inclination of the resultant deflection with the U-U axes. This resultant displacement is perpendicular to the neutral axis n-n (Fig. 3) but not in the plane of the load P. In Fig. the following notation has been used:-

$$O O' = D$$

$$O' P = D_v$$

$$O P = D_u$$

$$O' Q = D_x$$

$$O Q = D_y$$

$$\frac{\Delta_x = P \cos \phi \cdot L^3}{3EI_{uu}}$$

$$\tan \beta = \Delta_x / \Delta_y = O'P / OP = \dots\dots\dots$$

$$\Delta_y = P \sin \phi \cdot L^3$$

$$3EI_{vv}$$

$$= I_{VV} / I_{UU} \cot \phi \dots \dots \dots (vi)$$

For the angle section used in the present experiment I_{UU} and I_{VV} can be known from the tables of Bureau of Indian Standard hand book for properties of standard section. Therefore a given angle ϕ , the magnitude of angle β can be calculated from equation (vi).

The horizontal and vertical components of the deflection can be calculated on the basis of the geometry available as shown in Fig. 4. It can be seen.

$$\Delta_{\zeta} = \Delta \cdot \cos (\phi + \beta)$$

$$> \dots \dots \dots (vii)$$

$$\Delta_{\psi} = \Delta \cdot \sin (\phi + \beta)$$

Similarly

$$\begin{aligned} \Delta_{\zeta} &= \Delta \cdot (\cos \phi \cos \beta - \sin \phi \sin \beta) \\ &= \Delta_y \cos \phi \Delta_{\zeta} \sin \beta \dots \dots \dots (viii) \end{aligned}$$

$$\begin{aligned} \Delta_{\psi} &= \Delta \cdot (\sin \phi \cos \beta + \cos \phi \sin \beta) \\ &= \Delta_y \sin \phi \Delta_{\zeta} \cos \beta \dots \dots \dots (xi) \end{aligned}$$

Therefore the procedure of calculating the deflections would be:-

- Calculate Δ_U and Δ_V using equations (ii and iii)
- Compute Δ using equations (iv)
- Compute β using equation (vi)
- Calculate the required values of Δ_x Δ_y using equations (viii) and (ix).

Apparatus:-

Cantilever beam having an equal angle section. The beam is fixed at one end with possibility of rotation of 45° intervals and clamped. At the free end, the loading arrangements are such that vertical loading is always ensured, dial gauges, weights etc.

Procedure:-

- Clamp the beam at zero position and put a weight of 500gms (5N) on the hanger and take the zero loading on the beam to activate the member.
- Set the dial gauges to zero reading to measure vertical and horizontal displacements at the free end of the beam.
- Load the beam in steps of 1kg (10N) up to 8kg (80N) and note the vertical and horizontal deflections each time.
- Repeat the steps (a) to (c) turning the beam through 45° intervals. Problem of unsymmetrical

bending will arise only in those cases where the legs of the angle section are in horizontal and vertical positions. In those cases both vertical and horizontal deflections need be measured.

- Compute the theoretical deflections and compare with those measured experimentally.

Precautions:-

- (1) Take care to see that you do not exert force on the free end of the cantilever beam.
- (2) Put the load on the hanger gradually without any jerk.
- (3) Perform the test at a location which is free from vibrations.

Observations and calculations:-

- (a) Material of beam – mild steel.
- (b) Younges modulus of the material $(E) = 2 \times 10^6 \text{kg/cm}^2 (2 \times 10^5 \text{N/mm}^2)$
- (c) Span of cantilever beam $(L) = \text{cm}$
- (d) Sectional properties

- i. Size =
- ii. $I_{xx} = \text{mm}^4$
- iii. $I_{yy} = \text{mm}^4$
- iv. $I_{UU} = \text{mm}^4$
- v. $I_{VV} = \text{mm}^4$
- vi. Area = mm^2

Table (B-19.1) Deflections

Sr. No.	Angle	Load (kg) (N)	Observed Deflections (mm)		Measured Deflections (mm)	
			X	Y	x	y
1.	0					
2.	45°					
3.	90°					
4.	135°					
5.	180°					
6.	225°					
7.	270°					
8.	315°					

Results:-

EXPERIMENT NO :

DATE:

10. (A) INFLUENCE LINES FOR FIXED BEAM

EXPERIMENT NO :

DATE:

10.(B) INFLUENCE LINES FOR CONTINUOUS BEAM

EXPERIMENT NO :

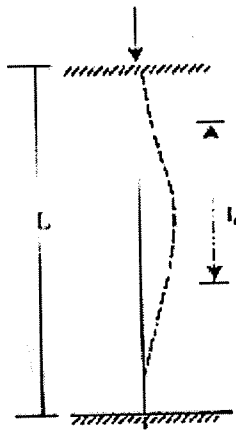
DATE:

11. COLUMNS AND STRUTS

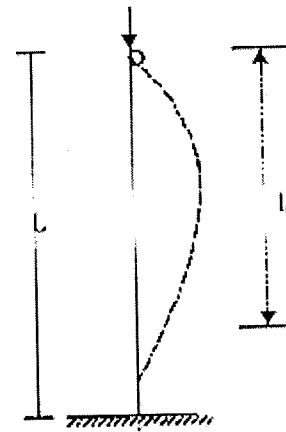
Aim: - To study behavior of different types of columns and find Euler's buckling load for each case.

Apparatus: - Column Buckling Apparatus, Weights, Hanger, Dial Gauge, Scale, Vernier caliper.

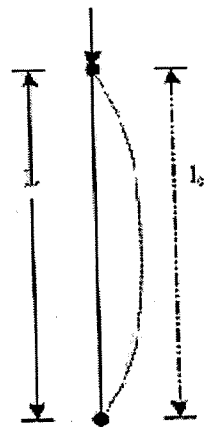
Diagram:-



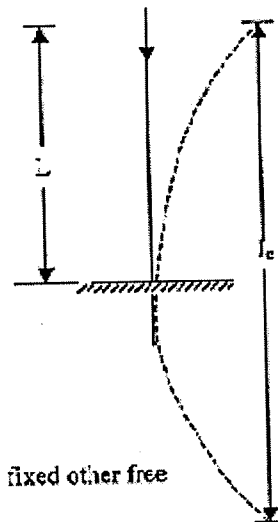
(i) Both ends fixed



(ii) One end fixed other pinned



(iii) Both ends pinned



(iv) One end fixed other free

Theory :-If compressive load is applied on a column, the member may fail either by crushing or by buckling depending on its material, cross section and length. If member is considerably long in comparison to its lateral dimensions it will fail by buckling. If a member shows signs of buckling the member leads to failure with small increase

in load. The load at which the member just buckles is called as crushing load. The buckling load, as given by Euler, can be found by using following expression.

$$P = \frac{\pi^2 EI}{l_e^2}$$

Where,

$$P = \frac{\pi^2 EI}{l_e^2}$$

E = Modulus of Elasticity = 2×10^5 N/mm² for steel

I = Least moment of inertia of column section

Le = Effective length of column

Depending on support conditions, four cases may arise. The effective length for each of which are given as:

1. Both ends are fixed $l_e = L/2$
2. One end is fixed and other is pinned $l_e = L/\sqrt{2}$
3. Both ends are pinned $l_e = L$
4. One end is fixed and other is free $l_e = 2L$

Procedure: -

- Pin a graph paper on the wooden board behind the column.
- Apply the load at the top of columns increasing gradually. At certain stage of loading the columns shows abnormal deflections and gives the buckling load.
- Note the buckling load for each of the four columns.
- Trace the deflected shapes of the columns over the paper. Mark the points of change of curvature of the curves and measure the effective or equivalent length for each case separately.
- Calculate the theoretical effective lengths and thus buckling loads by the expressions given above and compare them with the observed values.

Observation: -

- 1) Width of strip (mm) $b =$
- 2) Thickness of strip (mm) $t =$
- 3) Length of strip (mm) $L =$
- 4) Least moment of inertia

$$I = \frac{bt^3}{12}$$

Observation Table:-

Sr. No	End condition	Euler's Buckling load		Effective Length (mm)	
		Theoretical	Observed	Theoretical	Observed
1	Both ends fixed				
2	One end fixed and other pinned				
3	Both ends pinned				
4	One end fixed and other free.				

Sample Calculation: - End condition: Both ends fixed

$$\text{Euler's buckling load.} = P = \frac{\pi^2 EI}{l_e^2}$$

Effective Length (mm) =.

Result :-

The theoretical and experimental Euler's buckling load for each case is found nearly same

